EX35 $A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 3 & 1 & 1 \end{bmatrix}$

1) $\det A = \left|\begin{matrix} 3 & 0 & 2 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{matrix}\right| \begin{matrix} 3 & 0 \\ 2 & -1 \\ 1 & 0 \end{matrix}$

$= -3+0+0-(-2)-0-0 = -1$

2) $\det B = \left|\begin{matrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 3 & 1 & 1 \end{matrix}\right| \begin{matrix} 1 & -1 \\ 0 & 2 \\ 3 & 1 \end{matrix}$

$= 2+0+0-6-0-0 = -4$

3) $\det A^t = \det A = -1$

4) $\det B^t = \det B = -4$

5) $\det A^{-1} = \dfrac{1}{\det A} = \dfrac{1}{-1} = -1$

6) $\det B^{-1} = \dfrac{1}{\det B} = \dfrac{1}{-4} = -\dfrac{1}{4}$

7) $\det(AB) = (\det A)(\det B) = (-1)(-4) = 4$

8) $\det(2A) = 2^3 \det A = 8(-1) = -8$

9) $\det(3B) = 3^3 \det B = 27(-4) = -108$

10) $\det(A^{-1})^t = \det A^{-1} = -1$

11) $\det(2B^t)^{-1} = \dfrac{1}{\det(2B^t)} = \dfrac{1}{2^3 \det B^t} = \dfrac{1}{8 \det B} = \dfrac{1}{8(-4)} = \dfrac{-1}{32}$

12) $\det(2B^{-1})^t = \det(2B^{-1})$

$= 2^3 \det B^{-1} = 8(-\frac{1}{4}) = -2$

13) $\det(2A)(3B) = \det(6AB) = 6^3 \det(AB)$

$\frac{1}{\det A}$ ✓ $\quad \frac{1}{A}$ ✗

$\underbrace{3\times 3 \quad 3\times 3}$

$= 216 (\det A)(\det B)$

$= 216(-1)(-4)$

$= 864$

14) $\det(3A^t)(2B)^{-1}$

$= \det(3A^t)\ \det(2B)^{-1} = 3^3(\det A)\ \cancel{\det}\ \frac{1}{\det 2B}$

$= \frac{-27}{2^3 \det B} = \frac{-27}{8(-4)} = \frac{27}{32}$

15) $\det(A+B)$

$$A+B = \begin{bmatrix} 3 & 0 & 2 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 & 3 \\ 2 & 1 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

$$\det(A+B) = \begin{vmatrix} 4 & -1 & 3 \\ 2 & 1 & 1 \\ 4 & 1 & 2 \end{vmatrix} \begin{matrix} 4 & -1 \\ 2 & 1 \\ 4 & 1 \end{matrix}$$

$= 8 + (-4) + 6 - 12 - 4 - (-4)$

$= -2$

16) $\det(A^t - 2B)$

$$A^t - 2B = \begin{bmatrix} 3 & 0 & 2 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}^t - 2\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 0 \\ 2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -2 & 2 \\ 0 & 4 & 0 \\ 6 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & -1 \\ 0 & -5 & 0 \\ -4 & -1 & -1 \end{bmatrix}$$

$$\det(A^t-2B) = \left|\begin{matrix}1 & 4 & -1\\ 0 & -5 & 0\\ -4 & -1 & -1\end{matrix}\right|\begin{matrix}1 & 4\\ 0 & -5\\ -4 & -1\end{matrix}$$

$$= 5+0+0-(-20)-0-0$$

$$= 25$$

EX 3b

1) $A = \begin{bmatrix}1 & 0 & 2\\ 2 & -1 & 1\\ 1 & 0 & 1\end{bmatrix}\begin{matrix}1 & 0\\ 2 & -1\\ 1 & 0\end{matrix}$

$\det A = -1+0+0-(-2)-0-0 = 1$

2) $B = \begin{bmatrix}3 & 0 & 2\\ 2 & -1 & 1\\ 3 & -1 & 2\end{bmatrix}$ $\quad \det B = -6+0-4+6+3-0$

$= -1$

~~det B =~~

3) $C = \begin{bmatrix}-1 & 0 & 2\\ 0 & -1 & 1\\ -1 & 0 & 1\end{bmatrix}$ $\quad \det C = 1+0+0-2-0-0$

$= -1$

4) $D = \begin{bmatrix}3 & 0 & 2\\ 3 & -1 & 1\\ 1 & 0 & 1\end{bmatrix}$ $\quad \det D = -3+0+0+2-0-0$

$= -1$

5) $E = \begin{bmatrix}1 & 0 & 1\\ 3 & -1 & 1\\ 3 & 0 & 2\end{bmatrix}$ $\quad \det E = -(-1) = 1$

6) $F = \begin{bmatrix}3 & 2 & 0\\ 3 & 1 & -1\\ 1 & 1 & 0\end{bmatrix}$ $\quad \det F = -(-1) = 1$

Ex 37

$$\det A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$$

$$B = \begin{bmatrix} f & d & e \\ c & a & b \\ i & g & h \end{bmatrix} \quad \det B = ?$$

จาก

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$$

$$\begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix} = -4$$

$$\begin{vmatrix} d & f & e \\ a & c & b \\ g & i & h \end{vmatrix} = 4$$

$$\begin{vmatrix} f & d & e \\ c & a & b \\ i & g & h \end{vmatrix} = -4$$

$$C = \begin{bmatrix} i & \boxed{h} & g \\ f & \boxed{e} & d \\ c & \boxed{b} & a \end{bmatrix} \qquad \det C = ?$$

จาก $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4 \quad \therefore \begin{vmatrix} g & \boxed{h} & i \\ d & \boxed{e} & f \\ a & \boxed{b} & c \end{vmatrix} = -4$

$$\begin{vmatrix} i & h & g \\ f & e & d \\ c & b & a \end{vmatrix} = 4$$

<u>แถม</u> $D = \begin{bmatrix} 2a & -2d & 6g \\ b & -e & 3h \\ c & -f & 3i \end{bmatrix}$

จาก $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4 \quad \therefore \begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} = 4$

$$\begin{vmatrix} 2a & 2d & 2g \\ b & e & h \\ c & f & i \end{vmatrix} = 2(4) = 8$$

$$\begin{vmatrix} 2a & -2d & 6g \\ b & -e & 3h \\ c & -f & 3i \end{vmatrix} = \cancel{8}\ 8\overset{(-1)}{\cancel{(-2)}}(3) = -24$$