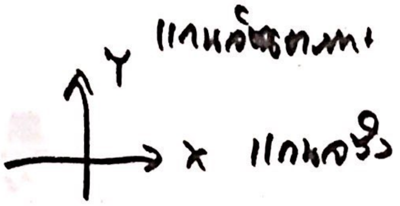
กราฟของจำนวนเชิงซ้อน

$z = a+bi = (a,b)$

$z = x+yi = (x,y)$ $\rightarrow$



6.37 $1 < |z-2-i| \le 4$

ให้ $z = (x,y) = x+yi$

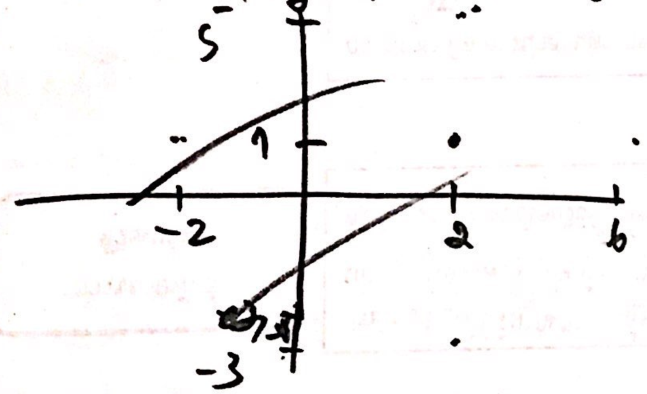
$1 < |x+yi-2-i| \le 4$

$1 < |(x-2)+(y-1)i| \le 4$

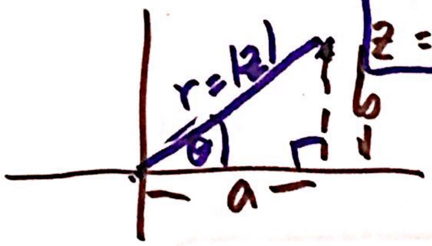
$1 < \sqrt{(x-2)^2+(y-1)^2} \le 4$

$1^2 < (x-2)^2+(y-1)^2 \le 4^2$

$1^2 < (x-2)^2+(y-1)^2$ และ $(x-2)^2+(y-1)^2 \le 4^2$



จำนวนเชิงซ้อนในรูปเชิงขั้ว (Polar Form)



$\boxed{z = a+bi} = (a,b)$

$\cos\theta = \frac{a}{r} \quad \therefore \quad a = r\cos\theta$

$\sin\theta = \frac{b}{r} \quad \therefore \quad b = r\sin\theta$

$\boxed{\tan\theta = \frac{b}{a}}$

จาก $z = a+bi$

$= r\cos\theta + (r\sin\theta)i$

✓ $\boxed{z = r(\cos\theta + i\sin\theta)}$ รูปเชิงขั้ว

✓ $\boxed{z = r\,\text{cis}\,\theta}$

$= r\angle\theta$

$$\begin{array}{r} 2+3i \\ +\;4-i \\ \hline 6+2i \end{array}$$

ทฤษฎีบท ให้ $z_1 = r_1\,\text{cis}\,\theta_1$

และ $z_2 = r_2\,\text{cis}\,\theta_2$

① $z_1 \cdot z_2 = r_1 r_2\,\text{cis}(\theta_1+\theta_2)$

② $\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1-\theta_2)$

③ $z_1^n = r_1^n\,\text{cis}(n\theta_1)$

แบบฝึกหัดทบทวน

7 ให้ $z_1 = -3+3i$, $z_2 = -2-2\sqrt{3}i$ จงหา

7.1) เปลี่ยนจำนวนเชิงซ้อนทั้งสองเป็นรูปเชิงขั้ว

จาก $z_1 = -3+3i$

$$r = \sqrt{(-3)^2+3^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$\tan\theta_1 = \frac{3}{-3} = -1 \qquad 45^\circ = \frac{\pi}{4}$$

$$\theta_1 = \frac{3\pi}{4}$$

$$z_1 = r_1 \operatorname{cis}\theta_1 = 3\sqrt{2}\operatorname{cis}\frac{3\pi}{4}$$

จาก $z_2 = -2-2\sqrt{3}i$

$$r_2 = \sqrt{(-2)^2+(-2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$$

$$\tan\theta_2 = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$$

$$\theta_2 = \frac{4\pi}{3}$$

$$z_2 = r_2 \operatorname{cis}\theta_2 = 4\operatorname{cis}\frac{4\pi}{3}$$

7.2) $z_1^6 \cdot z_2^8$

$$= \left(3\sqrt{2}\operatorname{cis}\frac{3\pi}{4}\right)^6 \cdot \left(4\operatorname{cis}\frac{4\pi}{3}\right)^8$$

$$= \left[(3\sqrt{2})^6 \operatorname{cis} 6\left(\frac{3\pi}{4}\right)\right]\left[4^8 \operatorname{cis} 8\left(\frac{4\pi}{3}\right)\right]$$

$$= (3\sqrt{2})^6(4^8)\operatorname{cis}\left(\frac{9\pi}{2}+\frac{32\pi}{3}\right) = 3^6\cdot 2^3\cdot 4^8 \operatorname{cis}\frac{91\pi}{6}$$

$$= 3^6\cdot 2^3\cdot 4^8\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$$

7.3) $\dfrac{z_1^5}{z_2^3}$ $\qquad z_1 = 3\sqrt{2}\,\text{cis}\,\dfrac{3\pi}{4}$

$\qquad z_2 = 4\,\text{cis}\,\dfrac{4\pi}{3}$

$$= \frac{\left(3\sqrt{2}\,\text{cis}\,\frac{3\pi}{4}\right)^5}{\left(4\,\text{cis}\,\frac{4\pi}{3}\right)^3} = \frac{(3\sqrt{2})^5\,\text{cis}\,5\left(\frac{3\pi}{4}\right)}{4^3\,\text{cis}\,3\left(\frac{4\pi}{3}\right)}$$

$$= \frac{3^5(4\sqrt{2})\,\text{cis}\,\frac{15\pi}{4}}{4^3\,\text{cis}\,4\pi} = \frac{3^5\sqrt{2}\,\text{cis}\,\frac{15\pi}{4}}{4^2\,\text{cis}\,4\pi}$$

4π $\qquad (1,0)$

$\frac{\pi}{4}$

$+,-$

$$= \frac{3^5\sqrt{2}\,\text{cis}\,\frac{15\pi}{4}}{4^2(1+0i)} = \frac{3^5\sqrt{2}}{16}\left(+\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$$

$$= \frac{3^5\cancel{\sqrt{2}}}{16}\left(\frac{\cancel{\sqrt{2}}}{\cancel{2}}\right)(1-i) = \frac{243}{16}(1-i)$$